Illinois Institute of Technology Homework 8

***Finding Invariants; Array Assignments***

*CS 536: Science of Programming Due Wed Nov 16 14 [Fixed 11/8]*

1. [15 = 5\*3 pts] Let *q* ≡ y ≥ 0 ∧ z = 2^y ≤ x < 2^(y+1) [x is constant, y is variable]. What are the five candidate invariants/loop test combinations you get from *q* if you replace a constant by a variable? (Not all the candidates make usable invariants.) If you can, include a range limitation on the new variable.

Ans: Below, the new variable is v in all 5 cases, but it's fine to use different variables. The ranges (v ≥ 0 or 1) are just educated guesses.  
• {**inv** v ≥ 0 ∧ y ≥ v ∧ z = 2^y ≤ x < 2^(y+1)} **while** v ≠ 0   
• {**inv** v ≥ 0 ∧ y ≥ 0 ∧ z = v^y ≤ x < 2^(y+1)} **while** v ≠ 2

• {**inv** v ≥ 0 ∧ y ≥ 0 ∧ z = 2^y ≤ v < 2^(y+1)} **while** v ≠ x  
• {**inv** v ≥ 0 ∧ y ≥ 0 ∧ z = 2^y ≤ x < v^(y+1)} **while** v ≠ 2  
• {**inv** v ≥ 1 ∧ y ≥ 0 ∧ z = 2^y ≤ x < 2^(y+v)} **while** v ≠ 1

1. [12 = 4\*3 pts] Rewrite *q* as (y ≥ 0) ∧ (z = 2^y) ∧ (2^y ≤ x) ∧ (x < 2^(y+1)). What are the four candidate invariant/loop test combinations you get if you drop a (single) conjunct from *q*?

Ans: With q ≡ (y ≥ 0) ∧ (z = 2^y) ∧ (2^y ≤ x) ∧ (x < 2^(y+1)), we get  
• {inv z = 2^y ≤ x < 2^(y+1)} while y < 0 …  
• {inv y ≥ 0 ∧ 2^y ≤ x <  2^(y+1)} while z ≠ 2^y …  
• {inv y ≥ 0 ∧ z = 2^y ∧ x < 2^(y+1)} while 2^y > x  …  
• {inv y ≥ 0 ∧ z = 2^y ≤ x} while x ≥ 2^(y+1)  …

For Problems 3 and 4, use *wp* to find a precondition *p* for the given program. Give a full proof outline. Use logical manipulations to simplify your predicates as you calculate them (if it makes your life easier).

1. [12 points] {*p*} b[i] := a; b[j] := c {b[i] ≤ b[j]}

Ans: For the full proof outline, we want p and q such that  
{p} b[i] := a; {q} b[j] := c {b[i] ≤ b[j]}  
We'll calculate q ⇔ wp(b[j] := c, b[i] ≤ b[j]) and p ⇔ wp(b[i] := a, q).   
q ≡ wp(b[j] := c, b[i] ≤ b[j])  
≡ (b[i] ≤ b[j])[ c/b[j] ]  
≡ (b[i])[ c/b[j] ] ≤ (b[j])[ c/b[j] ]  
≡ if i = j then c else b[i] fi ≤ c  
⇔ i = j ∨ b[i] ≤ c

p ≡ wp(b[i] := a, q)  
≡ q[ a/b[i] ]  
≡ (i = j ∨ b[i] ≤ y)[ a/b[i] ]  
≡ i = j ∨ a ≤ y

1. [11 points] {*p*} b[i] := b[j]; b[j] := b[k] {b[i] < b[k]}

Ans: Again, let’s get p and q with {p} b[i] := b[j]; {q} b[j] := b[k] {b[i] < b[k]}  
q ⇔ wp(b[j] := b[k], b[i] < b[k])  
≡ (b[i] < b[k])[ b[k]/b[j]]  
≡ (b[i])[ b[k]/b[j]] < (b[k])[ b[k]/b[j]]  
≡ if i = j then b[k] else b[i] fi  
<  if k = j then b[k] else b[k] fi  
⇔ if i = j then b[k] else b[i] fi < b[k]  
⇔ i ≠ j ∧ b[i] < b[k]  
p ⇔ wp(b[i] := b[j], q)  
≡ q[ b[j]/b[i]]  
≡ (i ≠ j ∧ b[i] < b[k])[ b[j]/b[i]]  
≡ i ≠ j ∧ (b[i])[ b[j]/b[i]] < (b[k])[ b[j]/b[i]]  
≡ i ≠ j ∧ b[j] < if k = i then b[j] else b[k] fi  
⇔ i ≠ j ∧ k ≠ i ∧ b[j] < b[k]